Self-Evaluation Test

Time Allowed : 1 hour 30 minutes] [*Maximum Marks* : 55

1. Show that the relation R in the set $\{1, 2, 3\}$, given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ is reflexive but not symmetric. 1
2. Show that the binary operation $* : R \to R$ given by $a * b = a + 2b$ is not commutative. 1
3. Let $f: N \to N$ defined by $f(x) = 3x$. Show that 'f' is not an onto function. 1
4. Let n be a fixed positive integer. Define a relation R in Z as follows : $\forall\ a, b \in Z$, $aRb \Leftrightarrow a - b$ is divisible by n. Show that R is an equivalence relation. 4
5. Show that $f : [-1, 1] \to R$, given by $f(x) = \frac{x}{x+2}$, $x \neq -2$ is one-one. Find the inverse of function $f : [-1, 1] \to R_f$. 4
6. Show that the number of equivalence relations in the set $\{1, 2, 3\}$ containing $(1, 2)$ and $(2, 1)$ is two. 4
7. A relation $R : N \to N$ is given by $R = \{(a, b) : b$ is divisible by $a\}$. Check whether R is an equivalence relation. 4
8. Show that the relation $R : N \to N$ defined by $(a, b)\ R\ (c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in N \times N$ is an equivalence relation. 4
9. Let R be the set of real numbers and $*$ be the binary operation defined on R as $a * b = a + b - ab$, $\forall\ a, b \in R$. Find the identity element with respect to binary operation $*$. 4
10. Let $*$ be a binary operation on N, given by $a * b =$ l.c.m. (a, b) for $a, b \in N$. Find : (*i*) $2 * 4$, (*ii*) $3 * 5$, (*iii*) Is '$*$' associative ? 4
11. If $A = \{a, b, c, d\}$ and $f = \{(a, b), (b, d), (c, a), (d, c)\}$, show that f is one-one from A onto A. Find f^{-1}. 4
12. Show that the function $f: R \to R$ defined by $f(x) = \frac{x}{x^2+1}$, $\forall\ x \in R$ is neither one-one nor onto. 4
13. Let relation R, on the set of natural numbers N is defined as follows : $R = \{(x, y) \in N \times N : 2x + y = 41\}$. Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive. 4
14. Show that $f : N \cup \{0\} \to N \cup \{0\}$ given by $f(n) = \begin{cases} n+1, & \text{if } n \text{ is even} \\ n-1, & \text{if } n \text{ is odd} \end{cases}$ is a bijective function. 6
15. Let $A = N \times N$ and let $*$ be a binary operation on A defined by $(a, b) * (c, d) = (ac, bd)$. Show that (*i*) $(A, *)$ is commutative (*ii*) $(A, *)$ is associative. Find the identity element, if any, in A. 6

ANSWERS

5. $f^{-1}(x) = \frac{2x}{1-x}$
7. not an equivalence relation
9. 0
10. (*i*) 4 (*ii*) 15 (*iii*) yes, associative
11. $f^{-1} = \{(b, a), (d, b), (a, c), (c, d)\}$
13. Domain = $\{1, 2, 3, 4, ..., 20\}$, Range = $\{1, 3, 5, 7, ..., 39\}$
Neither reflexive, nor symmetric, nor transitive.
15. $(1, 1)$

Self-Evaluation Test

Time Allowed : 1 hour 30 minutes] [*Maximum Marks* : 40

1. If $\sin\left\{\sin^{-1}\frac{1}{5}+\cos^{-1}x\right\}=1$, then find the value of x. 1
2. Find the value of, $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$. 1
3. Prove that, $\sin^{-1}x=\cos^{-1}\sqrt{1-x^2}$. 1
4. Evaluate, $\tan^{-1}(-\sqrt{3})$. 1
5. Represent, $\sin^{-1}(2ax\sqrt{1-a^2x^2})$, $-\frac{1}{\sqrt{2}}\le ax\le\frac{1}{\sqrt{2}}$ in the simplest form. 4
6. Write the function, $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $x\neq 0$ in the simplest form. 4
7. Find the value of, $\tan\left[\frac{1}{2}\left\{\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right\}\right]$, $|x|<1$, $y>0$, $xy<1$. 4
8. Prove that, $\sin^{-1}x+\sin^{-1}y=\sin^{-1}\left[x\sqrt{1-y^2}+y\sqrt{1-x^2}\right]$. 4
9. Find the value of, $\tan^{-1}\left[2\cos\left\{2\sin^{-1}\frac{1}{2}\right\}\right]$. 4
10. Show that $\sin^{-1}\frac{3}{5}-\sin^{-1}\frac{8}{17}=\cos^{-1}\frac{84}{85}$. 4
11. Prove that, $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}$, $x\in\left(0,\frac{\pi}{4}\right)$. 4
12. Write the function, $\cot^{-1}(\sqrt{1+x^2}+x)$ in the simplest form. 4
13. If $\cos^{-1}\frac{x}{2}+\cos^{-1}\frac{y}{3}=\theta$, then prove that $9x^2-12xy\cos\theta+4y^2=36\sin^2\theta$. 4

ANSWERS

1. $\frac{1}{5}$ 2. $\frac{5\pi}{6}$ 4. $-\frac{\pi}{3}$ 5. $2\sin^{-1}(ax)$ 6. $\frac{1}{2}\tan^{-1}x$

7. $\frac{x+y}{1-xy}$ 9. $\frac{\pi}{4}$ 12. $\frac{x}{2}$

Time Allowed : 1 hour 30 minutes] [*Maximum Marks* : 50

1. Find the sum of matrix $A = \begin{pmatrix} 2 & -1 \\ 4 & 6 \end{pmatrix}$ and its additive inverse. 1
2. For the matrix A, show that $A - A^T$ is a skew-symmetric matrix. 1
3. Given a matrix $A = [a_{ij}]$, $1 \le i \le 3$ and $1 \le j \le 3$, where $a_{ij} = i + 2j$. Write the element
   (*i*) a_{11} (*ii*) a_{32}
   (*iii*) a_{23} (*iv*) a_{34}. 4
4. If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $B = [2 \quad -3 \quad 4]$, find AB. 4
5. If $A = \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then find k if $A^2 = kA - 2I$. 4
6. Find a matrix X, such that $A + 2B + X = 0$, where $A = \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}$. 4
7. Find X and Y, given that $3X - Y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $X - 3Y = \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$. 4
8. If $A = \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix}$, find $f(A)$, where $f(x) = x^2 - 5x + 7$. 4
9. Find the inverse using elementary transformations, if exists, for the matrix $\begin{bmatrix} 8 & -4 \\ -2 & 1 \end{bmatrix}$. 4
10. Find the values of p and q such that $A^2 + pI = qA$, where $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$. 4
11. Find the value of x, $x \in I$ such that $[x \quad 4 \quad -1] \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 0 \\ 2 & 2 & 4 \end{bmatrix} [x \quad 4 \quad -1]^T = 0$. 4
12. Prove the following by principle of Mathematical Induction, if $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then,
    $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ for every positive integer n. 6
13. Using elementary transformations, find the inverse of matrix $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$. 6

Self-Evaluation Test

Time Allowed : 1 hour 30 minutes] [*Maximum Marks* : 55

1. If $\begin{vmatrix} 2x & 4 \\ -1 & x \end{vmatrix} = \begin{vmatrix} 6 & -3 \\ 2 & 1 \end{vmatrix}$, find x. 1

2. Using properties of determinants, show that $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$. 1

3. Without actual expansion, prove that : $\begin{vmatrix} 0 & 99 & -998 \\ -99 & 0 & 997 \\ 998 & -997 & 0 \end{vmatrix} = 0$. 1

4. Find the matrix A, such that $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. 4

5. Without expanding the determinant, prove that : $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$. 4

6. If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$. 4

7. Show that : $\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix} = (a^3 + b^3)^2$. 4

8. Find A^{-1}, if $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Also show that $A^{-1} = \dfrac{A^2 - 3I}{2}$. 4

9. Prove that $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x + a + b + c)$. 4

10. Prove that : $\begin{vmatrix} ab & -b^2 & bc \\ ca & bc & -c^2 \\ -a^2 & ab & ca \end{vmatrix} = 4a^2b^2c^2$. 4

11. Prove that : $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca).$ 4

12. Using properties of determinants, solve for x : $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0.$ 4

13. Using matrix method, solve the following system of linear equations : 6

$x+y+z=3;\ 2x-y+z=2;\ x-2y+3z=2$

14. Show that the following determinant vanishes : 4

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}.$$

15. Find the product of matrices $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and use it for solving 6

the equations : $x+y+2z=1,\ 3x+2y+z=7,\ 2x+y+3z=2.$

ANSWERS

1. $x = \pm 2$

4. $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

8. $A^{-1} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$

12. $x = 0, 3a$

13. $x = 1, y = 1, z = 1$

15. $AB = 4I$; $x = 2, y = 1, z = -1$